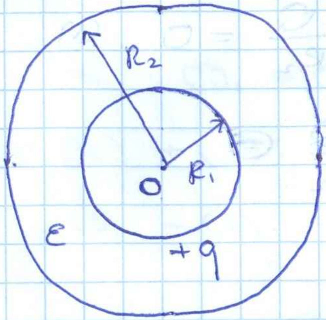
5.1.



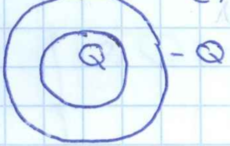

$$\varphi = \int_{R_1}^{\infty} E\,dr = \int_{R_1}^{R_2} \frac{kq}{\varepsilon r^2}\,dr + \int_{R_2}^{\infty} \frac{kq}{r^2}\,dr = \frac{-q}{4\pi\varepsilon\varepsilon_0}\frac{1}{r}\Big|_{R_1}^{R_2} + \frac{-q}{4\pi\varepsilon_0}\frac{1}{r}\Big|_{R_2}^{+\infty} =$$

$$= \frac{q}{4\pi\varepsilon\varepsilon_0}\left[\frac{1}{R_1} - \frac{1}{R_2}\right] + \frac{q}{4\pi\varepsilon_0}\frac{1}{R_2} =$$

$$= \frac{q}{4\pi\varepsilon_0\varepsilon}\left[\frac{1}{R_1} + \frac{\varepsilon - 1}{R_2}\right]$$

$$C = \frac{q}{\varphi} = \frac{4\pi\varepsilon\varepsilon_0}{\frac{1}{R_1} + \frac{\varepsilon-1}{R_2}} = \frac{4\pi\varepsilon\varepsilon_0 R_1}{1 + (\varepsilon-1)\frac{R_1}{R_2}}$$

5.2




(1) go

$$\varphi_1 = \frac{kQ}{R_1} - \frac{kQ}{R_2}$$

$$\varphi_2 = \frac{kQ}{R_2} - \frac{kQ}{R_2} = 0$$

$$W_{(1)} = \frac{1}{2}\left[Q\varphi_1 - Q\varphi_2\right] = \frac{Q}{2}\left[\frac{kQ}{R_1} - \frac{kQ}{R_2}\right] =$$

$$= \frac{kQ^2}{2}\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \frac{kQ^2}{2}\left[\frac{R_2 - R_1}{R_1 R_2}\right] = \frac{CU^2}{2}$$

(2) после


$$\varphi_1 = 0 = \frac{kQ'}{R_1} - \frac{kQ}{R_2} = 0 \quad (1)$$

$$\varphi_2 = \frac{kQ'}{R_2} - \frac{kQ}{R_2} \ominus$$

$$(1) \Rightarrow Q' = Q\frac{R_1}{R_2}$$

$$\ominus \frac{kQ}{R_2}\left[\frac{R_1}{R_2} - 1\right]$$

$$W_{(2)} = \frac{1}{2}\left[Q'\varphi_1 - Q\varphi_2\right] = \frac{-kQ^2}{2R_2}\left[\frac{R_1}{R_2} - 1\right] =$$

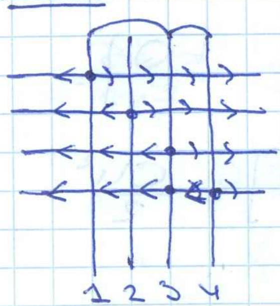
$$= \frac{kQ^2}{2} \cdot \frac{R_2 - R_1}{R_2 R_2}$$

$$\frac{W_{(2)}}{W_{(1)}} = \frac{R_1}{R_2} \Rightarrow W_{(2)} = \frac{R_1}{R_2} W_{(1)} = \frac{R_1}{R_2} \cdot \frac{CU^2}{2}$$

$$\Delta W = W_{(2)} - W_{(1)} = -\frac{CU^2}{2}\left(1 - \frac{R_1}{R_2}\right) =$$

$$= -2\pi\varepsilon_0 \frac{R_1 R_2}{R_2 - R_1} \cdot \frac{R_2 - R_1}{R_2} U^2 = -2\pi\varepsilon_0 R_1 U^2$$

$\boxed{5.3}$




$$Q_2 = Q \quad (1)$$

$$Q_1 + Q_3 + Q_4 = -Q \quad (2)$$

$$\varphi_3 - \varphi_4 = \frac{Q_1 + Q_2 + Q_3 - Q_4}{2\varepsilon_0 S} d_1 = 0 \Rightarrow$$

$$\Rightarrow \quad Q_1 + Q_2 + Q_3 - Q_4 = 0 \quad (3)$$

$$(1); (3) + (2): -\cancel{2}Q = \cancel{2}(Q_1 + Q_3) \quad (4)$$

$$(3) - (2): \quad Q_4 = 0$$

$$\varphi_1 - \varphi_3 = \frac{Q_1 - Q_2 - Q_3 - Q_4}{2\varepsilon_0 S} d_1 +$$

$$+ \frac{Q_1 + Q_2 - Q_3 - Q_4}{2\varepsilon_0 S} d = 0$$

$$(Q_1 - Q_3 - Q) d_1 + (Q_1 - Q_3 + Q) d = 0$$

$$(Q_1 - Q_3)(d_1 + d) = Q(d_1 - d)$$

$$\begin{cases} Q_1 - Q_3 = Q \dfrac{d_1 - d}{d_1 + d} \\ Q_1 + Q_3 = -Q \end{cases}$$

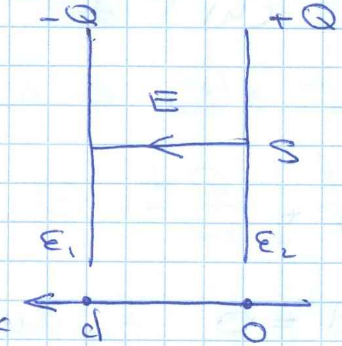
$$\begin{cases} 2Q_1 = -Q\left(\dfrac{d - d_1}{d + d_1} + 1\right) = -Q \dfrac{2d}{d + d_1} \\ 2Q_3 = -Q\left(1 - \dfrac{d - d_1}{d + d_1}\right) = -Q \dfrac{2d_1}{d + d_1} \end{cases}$$

$$U = \varphi_2 - \varphi_3 = \frac{Q_1 + Q_2 - Q_3 - Q_4}{2\varepsilon_0 S} d =$$

$$= \frac{d}{2\varepsilon_0 S} \cdot \left[\frac{-d + d_1}{(d + d_1)} Q + Q\right] = \frac{d}{2\varepsilon_0 S} \cdot \frac{2d_1}{(d + d_1)} Q$$

$$C = \frac{Q}{\varphi} = \frac{2\varepsilon_0 S(d+d_1)}{2dd_1} = \{d = 2d_1\} = \frac{3\varepsilon_0 S}{d} = 3C_0.$$

$\boxed{5.4}$




$$\begin{cases} \varepsilon(x) = \alpha x + \beta \\ \varepsilon(0) = \varepsilon_2 \qquad \beta = \varepsilon_2 \\ \varepsilon(d) = \varepsilon_1 \qquad \alpha = \dfrac{\varepsilon_1 - \varepsilon_2}{d} \end{cases}$$

$$\varepsilon(x) = \frac{\varepsilon_1 - \varepsilon_2}{d} x + \varepsilon_2$$

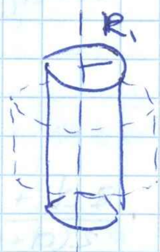
$$U = \int_0^d \frac{Q}{\varepsilon(x)\varepsilon_0 S} dx = \frac{Q}{\varepsilon_0 S} \int_0^d \frac{1}{\alpha x + \beta} dx =$$

$$= \frac{Q}{\varepsilon_0 S} \frac{\ln(\alpha x + \beta)}{\alpha} \Big|_0^d = \frac{Q}{\alpha \varepsilon_0 S} \left[\ln \varepsilon_1 - \ln \varepsilon_2\right] =$$

$$= \frac{Q}{\varepsilon_0 S} \frac{d}{\varepsilon_1 - \varepsilon_2} \ln \frac{\varepsilon_1}{\varepsilon_2}$$

$$C = \frac{Q}{U} = \frac{(\varepsilon_1 - \varepsilon_2)\varepsilon_0 S}{d \ln \frac{\varepsilon_1}{\varepsilon_2}}$$

$\boxed{5.5}$




по т Гаусса:

$$\oint D\, dS = 2\pi R_1 \cdot \ell \cdot \sigma$$

$$\varepsilon\varepsilon_0 E \cdot 2\pi r \cdot \cancel{\ell} = 2\pi R_1 \cdot \cancel{\ell} \cdot \sigma$$

$$E = \frac{\sigma R_1}{\varepsilon \varepsilon_0 r}$$

$$U = \varphi_1 - \varphi_2 = \int_{R_1}^{R_2} E\,dr = \frac{\sigma R_1}{\varepsilon\varepsilon_0}\int_{R_1}^{R_2}\frac{1}{r}\,dr = \frac{\sigma R_1}{\varepsilon\varepsilon_0}\ln\frac{R_2}{R_1}$$

т.о. а) при изменении радиусов $U = const$

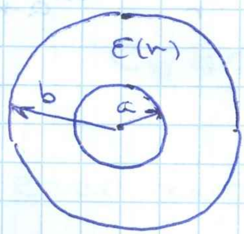
б) ~~двет~~ $E_1 = \frac{\sigma_1 \cancel{R_1}}{\varepsilon\varepsilon_0 \cancel{R_1}}$ $\quad E_2 = \frac{\sigma_2 \cancel{2R_1}}{\varepsilon\varepsilon_0 \cancel{2R_1}}$

$$\sigma_1 = \frac{dQ}{2\pi R_1 \cdot l} \qquad \sigma_2 = \frac{dQ}{4\pi R_1 \cdot l} \Rightarrow \sigma_2 = \frac{\sigma_1}{2}$$

$\Rightarrow E_2 = \frac{E_1}{2}$ (уменьшится вдвое).

$\boxed{5.6}$

$U = const \qquad \varepsilon = \frac{\varepsilon_\alpha a}{r}$




$$E = \frac{Q}{4\pi\varepsilon_0\,\varepsilon(r)\,r^2}$$

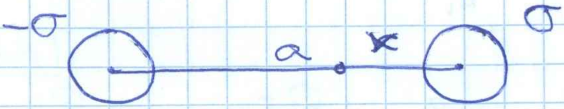
$$U = \int_a^b E(r)\,dr = \frac{Q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{\varepsilon(r)\,r^2} =$$

$$= \frac{Q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{\varepsilon_\alpha a r} = \frac{Q}{4\pi\varepsilon_0\varepsilon_\alpha a}\ln\frac{b}{a}$$

$$C = \frac{Q}{U} = \frac{4\pi\varepsilon_0\varepsilon_\alpha a}{\ln\left(\frac{b}{a}\right)}$$

$$W = \frac{CU^2}{2} = \frac{2\pi\varepsilon_0\varepsilon_\alpha a}{\ln(b/a)}U^2$$

## 5.7




$$E(x) = \frac{\sigma r}{\varepsilon_0 x} + \frac{\sigma r}{\varepsilon_0 (a-x)} \quad (\text{см. } 5.5)$$

$$U = \varphi_1 - \varphi_2 = \int_r^{a-r} E(x)\,dx = \frac{\sigma r}{\varepsilon_0} \int_r^{a-r} \left[\frac{1}{x} + \frac{1}{a-x}\right] dx$$

$$= \frac{\sigma r}{\varepsilon_0} \left[\ln x - \ln(a-x)\right]\Big|_r^{a-r} =$$

$$= \frac{\sigma r}{\varepsilon_0} \ln \frac{x}{a-x}\Big|_r^{a-r} = \frac{\sigma r}{\varepsilon_0}\left[\ln\frac{a-r}{r} - \ln\frac{r}{a-r}\right] =$$

$$= \frac{2\sigma r}{\varepsilon_0} \ln \frac{a-r}{r}$$

$$C = \frac{2\pi r \cdot h \sigma}{2\sigma r} \cdot \frac{\varepsilon_0}{\ln\frac{a-r}{r}} \approx \frac{\pi \varepsilon_0 h}{\ln\frac{a}{r}}$$

$$C_h = \frac{\pi \varepsilon_0}{\ln \frac{a}{r}}$$

## 5.9

а) не изменилась (физ. хар-ка)

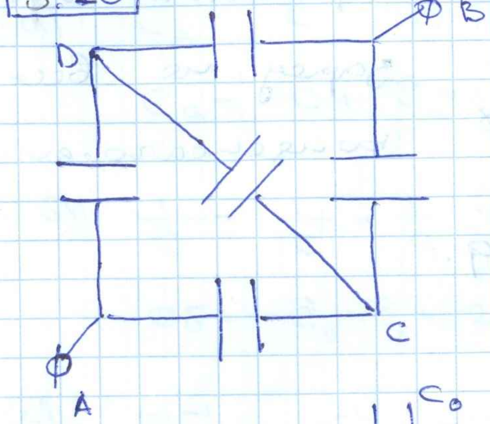
б) 1) $U_0 = \mathcal{E} = \frac{nq}{C}$

2) $U = \frac{(n-1)q}{C}$

$\Rightarrow \frac{U}{U_0} = \frac{n-1}{n} < 1$

уменьшилась.

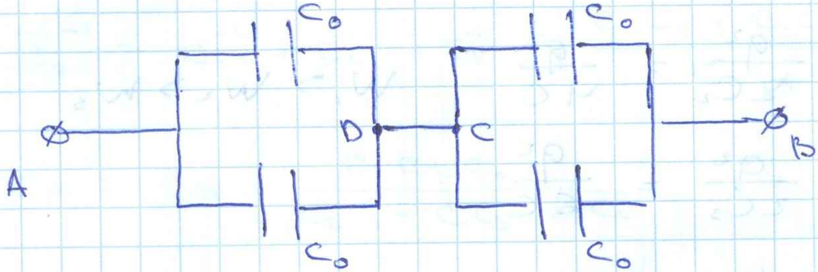
$\boxed{5.10}$




В силу симметрии задачи

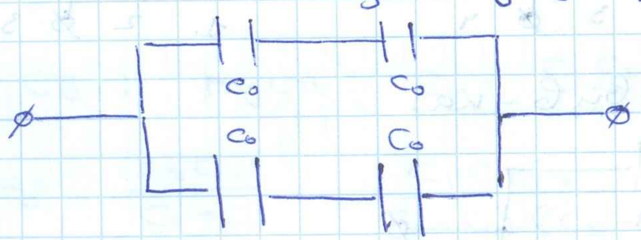
$$\varphi_D = \varphi_C$$

поэтому схема эквив-на схеме:




$C_{\parallel} = 2C_0$ ; $\quad \frac{1}{C_\Sigma} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C} \Rightarrow C_\Sigma = C_0.$

если ключ К не замкнут, то:




$\frac{1}{C} = \frac{1}{C_0} + \frac{1}{C_0} = \frac{2}{C_0}$ ; $\quad C_\Sigma = 2C = C_0.$

$\boxed{5.11}$ В силу закона сохранения заряда

заряд на всех конденсаторах будет одинаковый $q$.

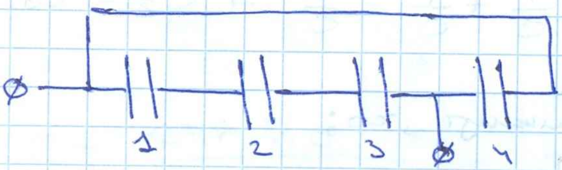
$$W_1 = \frac{q^2}{2C_1} = \frac{q^2}{2C}$$

$$W_2 = \frac{q^2}{2C_2} = \frac{q^2}{4C} \qquad W_1 > W_2 > W_3$$

$$W_3 = \frac{q^2}{2C_3} = \frac{q^2}{6C}$$

$\boxed{5.12}$ 1

а)




б)

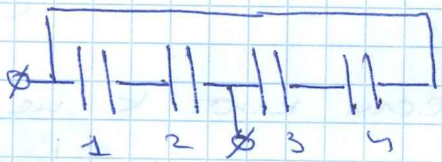



схема а) эквив-на:

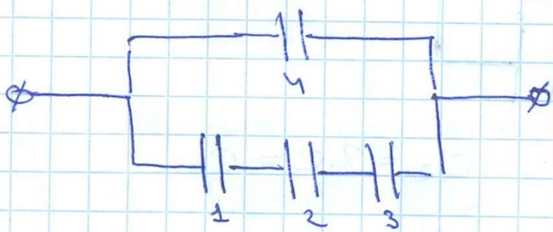
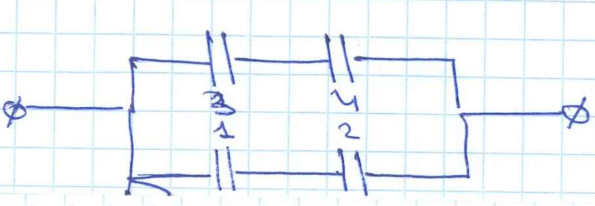



схема б) эквив-на:



а) $\frac{1}{\hat{C}_a} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C} \Rightarrow \hat{C} = \frac{C}{3}$

$C_a = C + \hat{C}_a = \frac{4C}{3}$

б) $\frac{1}{\hat{C}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C} \Rightarrow \hat{C} = \frac{C}{2}$

$C_б = \hat{C}_б + \hat{C}_б = 2\hat{C} = C$ $\qquad C_б < C_a$

② а) $\hat{C}_a^{-1} = C_1^{-1} + C_2^{-1} + C_3^{-1} = \frac{C_2C_3 + C_1C_3 + C_1C_2}{C_1C_2C_3}$

$C_a = C_4 + \frac{C_1C_2C_3}{C_2C_3 + C_1C_3 + C_1C_2}$

б) $\hat{C}_{б_1}^{-1} = C_1^{-1} + C_2^{-1} = \frac{C_1 + C_2}{C_1C_2}$ $\qquad \hat{C}_{б_2}^{-1} = \frac{C_3 + C_4}{C_3C_4}$

$C_б = \frac{C_1C_2}{C_1 + C_2} + \frac{C_3C_4}{C_3 + C_4}$

$C_a = C_б$ : пусть $C_4 = \frac{C_1C_2}{C_1 + C_2}$

тогда $\frac{C_3C_4}{C_3 + C_4} = \frac{C_1C_2C_3}{(C_1 + C_2)\left(C_3 + \frac{C_1C_2}{C_1 + C_2}\right)} =$

$= \frac{C_1C_2C_3}{C_1C_3 + C_2C_3 + C_1C_2}$

$\boxed{5.13}$

$$\omega_{E_1} = \frac{W_1}{V} = \frac{1}{2}\varepsilon\varepsilon_0 E_1^2$$

$$\omega_{E_2} = \frac{W_2}{V} = \frac{1}{2}\varepsilon_0 E_2^2$$

$$D = E_1 \varepsilon\varepsilon_0 = E_2 \varepsilon_0 = const$$

$$\frac{W_1}{W_2} = \frac{\varepsilon E_1^2}{E_2^2} = \frac{\varepsilon E_1^2}{\varepsilon^2 E_1^2} = \frac{1}{\varepsilon} \Rightarrow W_2 = \varepsilon W_1$$

$\boxed{5.14}$

$R_1 = 10 \text{ см} = 10^{-2}$ м $\qquad \varphi_1 = 450$ В

$R_2 = 20 \text{ см} = 2\cdot 10^{-2}$ м $\qquad \varphi_2 = 300$ В

$R_3 = 30 \text{ см} = 3\cdot 10^{-3}$ м $\qquad \varphi_3 = 150$ В

$$\frac{kq_i}{R_i} = \varphi_i \Rightarrow q_i = \frac{\varphi_i R_i}{k}$$

$$W_1 = \frac{1}{2}\sum_{i=1}^{3} q_i\varphi_i = \frac{1}{2k}\sum_{i=1}^{3}\varphi_i^2 R_i$$

---

$$\varphi_0 = \frac{kq_i'}{R_i} \qquad q_i' = \varphi_0 \cdot \frac{R_i}{k} = q_1' \frac{R_i}{R_1}$$

$$W_2 = \frac{1}{2}\sum_{i=1}^{3} q_i'\varphi_0 = \frac{1}{2}\cdot\frac{kq_1'}{R_1}\sum_{i=1}^{3} q_i' = \frac{kq_1'}{2R_1}\sum_{i=1}^{3} q_1'\frac{R_i}{R_1} =$$

$$= \frac{kq_1'^2}{2R_1^2}\sum_{i=1}^{3} R_i \; (=)$$

$$q_1' + q_2' + q_3' = q_1 + q_2 + q_3$$

$$q_1' + q_1' \frac{R_2}{R_1} + q_1' \frac{R_3}{R_1} = \sum_{i=1}^{3} \frac{\varphi_i R_i}{k}$$

$$q_1' \frac{R_1 + R_2 + R_3}{R_1} = \frac{1}{k} \sum_{i=1}^{3} \varphi_i R_i$$

$$q_1' = \frac{R_1}{R_1 + R_2 + R_3} \cdot \frac{1}{k} \sum_{i=1}^{3} \varphi_i R_i = \frac{R_1}{\sum R_i} \frac{1}{k} \sum \varphi_i R_i$$

$$= \quad \frac{k \sum R_i}{2 R_1^2} q_1'^2 = \frac{1}{2k \sum R_i} \left( \sum \varphi_i R_i \right)^2 =$$

$$= \frac{1}{2k} \cdot \frac{1}{R_1 + R_2 + R_3} \cdot \Big[ \varphi_1^2 R_1^2 + \varphi_2^2 R_2^2 + \varphi_3^2 R_3^2 +$$

$$+ 2 \varphi_1 \varphi_2 R_1 R_2 + 2 \varphi_2 \varphi_3 R_2 R_3 + 2 \varphi_1 \varphi_3 R_1 R_3 \Big]$$

$$W_1 = \frac{1}{2k} \cdot \frac{1}{R_1 + R_2 + R_3} \cdot \Big[ \varphi_1^2 R_1^2 + \varphi_1^2 R_1 R_2 + \varphi_1^2 R_1 R_3 +$$

$$+ \varphi_2^2 R_2^2 + \varphi_2^2 R_1 R_2 + \varphi_2^2 R_2 R_3 +$$

$$+ \varphi_3^2 R_3^2 + \varphi_3^2 R_1 R_3 + \varphi_3^2 R_2 R_3 \Big]$$

$$\Delta W = W_2 - W_1 = \frac{1}{2k} \cdot \frac{1}{R_1 + R_2 + R_3} \cdot$$

$$\cdot \Big[ (\varphi_1 - \varphi_2)^2 R_1 R_2 + (\varphi_2 - \varphi_3)^2 R_2 R_3 + (\varphi_1 - \varphi_3)^2 R_1 R_3 \Big]$$

$\boxed{5.15}$ (1) $U = const = \mathcal{E}$

$$W_1 = \frac{C_1 U^2}{2}; \quad C_1 = \frac{\varepsilon \varepsilon_0 S}{d}; \quad Q_1 = \frac{\varepsilon \varepsilon_0 S}{d} U$$

$$W_2 = \frac{C_2 U^2}{2}; \quad C_2 = \frac{\varepsilon_0 S}{d}; \quad Q_2 = \frac{\varepsilon_0 S}{d} U$$

$$W_1 + \mathcal{E} \Delta Q + A_{\text{мех}} = W_2$$

$$\Delta W = W_2 - W_1 = (1-\varepsilon) \frac{\varepsilon_0 S}{2d} U^2$$

$$A_{\text{мех}} = \Delta W - U \Delta Q = \Delta W - U(1-\varepsilon) \frac{\varepsilon_0 S}{d} U =$$

$$= \Delta W - 2\Delta W = -\Delta W$$

(2) $Q = const$

$$W_1 = \frac{Q^2}{2C_1}; \quad C_1 = \frac{\varepsilon \varepsilon_0 S}{d} \quad Q = C_1 U$$

$$W_2 = \frac{Q^2}{2C_2}; \quad C_2 = \frac{\varepsilon_0 S}{d}$$

$$W_1 + A_{\text{мех}} = W_2$$

$$A_{\text{мех}} = \Delta W = W_2 - W_1 = \frac{Q^2 d}{2\varepsilon_0 S}\left(1 - \frac{1}{\varepsilon}\right) =$$

$$= \frac{C_1^2 U^2}{2C_2}\left(1 - \frac{1}{\varepsilon}\right) = \frac{\varepsilon^2 \varepsilon_0 S}{2d} U^2 \left(1 - \frac{1}{\varepsilon}\right) =$$

$$= \frac{\varepsilon(\varepsilon - 1)\varepsilon_0 S}{2d} U^2$$

$\boxed{5.16}$

$$W_1 = \frac{q^2}{2C_1} = \frac{q^2 d_0}{2\varepsilon_0 S}$$

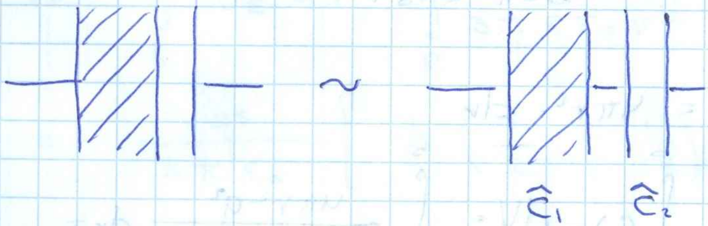
$$W_2 = \frac{q^2}{2C_2} = \frac{q^2 d}{2\varepsilon_0 S}$$

$$W_1 + A_{мех} = W_2 \Rightarrow A_{поля} = -A_{мех} = W_1 - W_2 =$$

$$= \frac{q^2}{2\varepsilon_0 S}(d_0 - d)$$

$\boxed{5.17}$ $U = const = \mathcal{E}$

$$W_1 = \frac{C_1 U^2}{2} = \frac{\varepsilon\varepsilon_0 S}{2d} U^2$$



$$\hat{C}_1 = \frac{\varepsilon\varepsilon_0 S}{d} \qquad \hat{C}_2 = \frac{\varepsilon_0 S}{x} \qquad W_2 = \frac{C_2 U^2}{2}$$

$$\frac{1}{C_2} = \frac{1}{\varepsilon_0 S}\left(\frac{d}{\varepsilon} + x\right) = \frac{1}{\hat{C}_1} + \frac{1}{\hat{C}_2} \Rightarrow C_2 = \frac{\varepsilon\varepsilon_0 S}{d + \varepsilon x}$$

$$W_1 + A_{мех} = W_2 \ ; \quad \frac{W_2}{W_1} = \frac{d}{d + \varepsilon x}$$

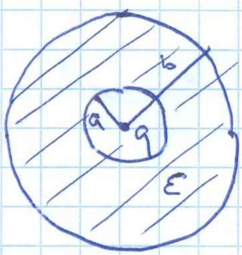
$$A_{мех} = W_2 - W_1 = W_2 - \frac{d + \varepsilon x}{d} W_2 = -\frac{\varepsilon x}{d} W_2$$

$$A = |A_{мех}| = \frac{\varepsilon x}{d} W_2 = \frac{\varepsilon x}{d} \frac{\varepsilon\varepsilon_0 S}{2(d + \varepsilon x)} U^2$$

$$A = \frac{\varepsilon \varepsilon_0 S U^2}{2d\left(\frac{d}{\varepsilon x}+1\right)} \Rightarrow \frac{d}{\varepsilon x} = \frac{\varepsilon \varepsilon_0 S U^2}{2dA} - 1$$

$$x = \frac{d}{\varepsilon}\,\frac{2Ad}{\varepsilon\varepsilon_0 S U^2 - 2Ad} = \frac{d}{\varepsilon}\cdot\frac{1}{\left(\frac{\varepsilon\varepsilon_0 S U^2}{2Ad}-1\right)}$$

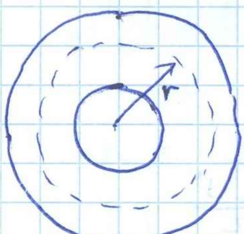
$\boxed{5.18}$




$$E = \frac{kq}{\varepsilon r^2} = \frac{q}{4\pi\varepsilon\varepsilon_0 r^2}$$

$$\omega_E = \frac{1}{2}\varepsilon\varepsilon_0 E^2 = \frac{\varepsilon\varepsilon_0}{2}\cdot\frac{q^2}{16\pi^2\varepsilon^2\varepsilon_0^2 r^4} =$$

$$= \frac{q^2}{32\pi^2\varepsilon\varepsilon_0 r^4}$$



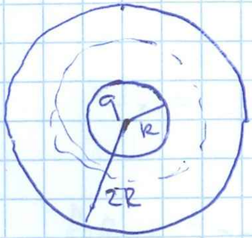

$$dV = 4\pi r^2 \cdot dr$$

$$W = \int_a^b \omega_E \, dV = \int_a^b \frac{4\pi r^2 q^2}{32\pi^2 \varepsilon\varepsilon_0 r^4}\, dr =$$

$$= \int_a^b \frac{q^2}{8\pi\varepsilon\varepsilon_0 r^2}\, dr = \left.\frac{-q^2}{8\pi\varepsilon\,\varepsilon_0 r}\right|_a^b =$$

$$= + \frac{q^2}{8\pi\varepsilon_0\varepsilon}\left[\frac{1}{a} - \frac{1}{b}\right] = \frac{q^2(b-a)}{8\pi\varepsilon\varepsilon_0 ab}$$

$\boxed{5.19}$



$$E = \begin{cases} \dfrac{q}{4\pi\varepsilon\varepsilon_0 r^2}, & R < r < 2R \\ \dfrac{q}{4\pi\varepsilon_0 r^2}, & r > 2R \end{cases}$$

$$\omega_E = \begin{cases} \dfrac{\varepsilon\varepsilon_0}{2} \cdot \dfrac{q^2}{16\pi^2\varepsilon^2\varepsilon_0^2 r^4} = \dfrac{q^2}{32\pi^2\varepsilon\varepsilon_0 r^4}; & R < r < 2R \\ \dfrac{\varepsilon_0}{2} \cdot \dfrac{q^2}{16\pi^2\varepsilon_0^2 r^4} = \dfrac{q^2}{32\pi^2\varepsilon_0 r^4}; & r > 2R \end{cases}$$

$$dV = 4\pi r^2 dr$$

$$W = \int_R^{+\infty} \omega_E \, dV = \int_R^{2R} \frac{q^2}{8\pi\varepsilon\varepsilon_0 r^2} dr + \int_{2R}^{+\infty} \frac{q^2}{8\pi\varepsilon_0 r^2} dr =$$

$$= \frac{q^2}{8\pi\varepsilon_0} \left[ \frac{1}{\varepsilon} \left( \frac{1}{R} - \frac{1}{2R} \right) + \frac{1}{2R} \right] =$$

$$= \frac{q^2}{8\pi\varepsilon_0} \left[ \frac{1}{2\varepsilon R} + \frac{1}{2R} \right] = \frac{q^2(\varepsilon + 1)}{16\pi\varepsilon\varepsilon_0 R}$$

$\boxed{5.20}$ (1) $\rho = \dfrac{q}{\frac{4}{3}\pi r^3}$

$$E(x) \cdot 4\pi x^2 = \frac{\rho \cdot \frac{4}{3}\pi x^3}{\varepsilon_0} \quad (\text{т. Гаусса}), \quad x < r$$

$$E(x) = \begin{cases} \dfrac{\rho x}{3\varepsilon_0} = \dfrac{q x}{4\pi r^3 \varepsilon_0}, & x < r \\ \dfrac{q}{4\pi\varepsilon_0 x^2}, & x \geq r \end{cases}$$

$$w_E = \frac{\varepsilon_0 E^2}{2} = \begin{cases} \dfrac{q^2 x^2}{32\pi^2 r^6 \varepsilon_0}, & x < r \\ \dfrac{q^2}{32\pi^2 \varepsilon_0 x^4}, & x \geq r \end{cases}$$

$$dV = 4\pi x^2 \, dx$$

$$W_1 = \int_0^{+\infty} w_E \, dV = \int_0^r \frac{q^2 x^4}{8\pi \varepsilon_0 r^6} dx + \int_r^{+\infty} \frac{q^2}{8\pi \varepsilon_0 x^2} dx =$$

$$= \frac{q^2}{40\pi \varepsilon_0 r^6} \cdot r^5 + \frac{q^2}{8\pi \varepsilon_0 r}$$

(2) $$W_2 = \frac{1}{2} q \frac{q}{4\pi \varepsilon_0 r} = \frac{q^2}{8\pi \varepsilon_0 r}$$

$$W_1 + A_{вн} = W_2 \Rightarrow A_{поле} = -A_{вн} = W_1 - W_2 =$$

$$= \frac{q^2}{40\pi \varepsilon_0 r}$$

$\boxed{5.21}$ $$C_1 = \frac{\varepsilon_0 \hat{S}}{d} \; ; \; C_2 = \frac{\varepsilon_0 \hat{S}}{d-s}$$

(1) $$W_1 = \frac{\varepsilon_0 \hat{S}}{2d} U^2 \qquad Q_1 = \frac{\varepsilon_0 \hat{S}}{d} U$$

$$W_2 = \frac{\varepsilon_0 \hat{S}}{2(d-s)} U^2 \qquad Q_2 = \frac{\varepsilon_0 \hat{S}}{d-s} U$$

$$A_{вн_1} = W_2 - W_1 - \mathcal{E} \Delta Q = U^2 \frac{\varepsilon_0 \hat{S}}{2} \left[ \frac{1}{d-s} - \frac{1}{d} - \left[ \frac{2}{d-s} - \frac{2}{d} \right] \right] = \frac{\varepsilon_0 \hat{S}}{2} U^2 \left[ \frac{1}{d} - \frac{1}{d-s} \right] = - \frac{\varepsilon_0 \hat{S} s}{2(d-s) d} U^2$$

(2) $W_1 = \frac{\varepsilon_0 \hat{S}}{2d} U^2 \qquad Q = \frac{\varepsilon_0 \hat{S}}{d} U$

$W_2 = \frac{Q^2}{2C_2} = \frac{\varepsilon_0^2 \hat{S}^2 U^2}{2d^2} \cdot \frac{d-s}{\cancel{\varepsilon_0 \hat{S}}} = \frac{\varepsilon_0 \hat{S}(d-s)}{2d^2} U^2$

$A_{вн2} = W_2 - W_1 = \frac{\varepsilon_0 \hat{S}}{2d} U^2 \left( \frac{d-s}{d} - 1 \right) = -\frac{\varepsilon_0 \hat{S} s}{2d^2} U^2$

$\frac{A_1}{A_2} = \frac{-A_{вн1}}{-A_{вн2}} = \frac{d}{d-s}$

$\boxed{5.22}$ из 5.5 известно:

$E(r) = \frac{\sigma R}{\varepsilon \varepsilon_0 r}, \quad r \in [R, 4R]$

$w_E(r) = \frac{\varepsilon \varepsilon_0 E^2}{2} = \frac{\sigma^2 R^2}{2 \varepsilon \varepsilon_0 r^2}, \quad R < r < 4R$

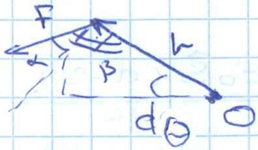
$dV = 2\pi r \cdot \ell \cdot dr$

$W_1 = \int_R^{2R} \frac{\sigma^2 R^2}{\cancel{2} \varepsilon \varepsilon_0 \cancel{r^2}} \cdot \cancel{2} \pi \cancel{r} \ell \, dr = \frac{\sigma^2 R^2 \pi \ell}{\varepsilon \varepsilon_0} \ln \frac{2R}{R}$

$W_2 = \int_{2R}^{4R} \frac{\sigma R^2}{2 \varepsilon \varepsilon_0 r^2} \cdot 2\pi r \ell \, dr = \frac{\sigma^2 R^2 \pi \ell}{\varepsilon \varepsilon_0} \ln \frac{4R}{2R}$

$\frac{W_1}{W_2} = \frac{1}{1}$

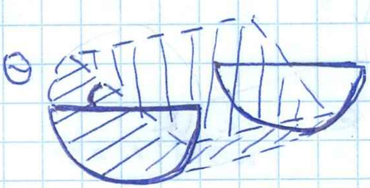
$\boxed{5.24}$ 1) Установим соотношение между работой и моментом силы.



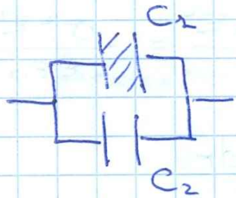

$$dA = F\cos\alpha \cdot r\,d\theta =$$
$$= F\,\cancel{\sin}\cos(\cancel{\alpha}\beta+\tfrac{\pi}{2})\,r\,d\theta =$$
$$= F r \sin\beta\,d\theta$$

$A = \int M\,d\theta$, тогда можно записать следующее

2) $$M = \frac{\partial W}{\partial \theta} = \frac{1}{2}U^2\frac{\partial C}{\partial \theta}$$




конденсатор можно разбить на две




$$C_1 = \frac{\varepsilon\varepsilon_0 S_1}{d} = \frac{\varepsilon\varepsilon_0(\pi-\theta)R^2}{2d}$$

$$C_2 = \frac{\varepsilon_0 S_2}{d} = \frac{\varepsilon_0\,\theta R^2}{2d}$$

$$C = C_1 + C_2 = \frac{\varepsilon_0\theta R^2}{2d} + \frac{\varepsilon\varepsilon_0(\pi-\theta)R^2}{2d}$$

$$\frac{\partial C}{\partial\theta} = \frac{\varepsilon_0 R^2}{2d} - \frac{\varepsilon\varepsilon_0 R^2}{2d} = (1-\varepsilon)\frac{\varepsilon_0 R^2}{2d}$$

$$M = \frac{1}{4}\frac{\varepsilon_0 R^2}{d}U^2(1-\varepsilon) < 0$$ (поле пытается втянуть диэлектр.)

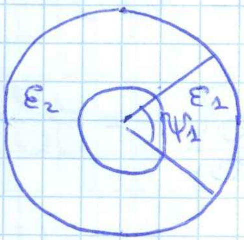
## 5.11 (продолжение)

3 конденсатора можно заменить одним

$$\frac{1}{C_0} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{3C} = \frac{11}{6C}$$

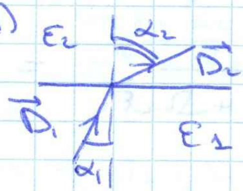
$$q = UC_0 = \frac{6}{11}UC$$

## 5.8




Рассмотрим поле на границе диэлектриков и металла:

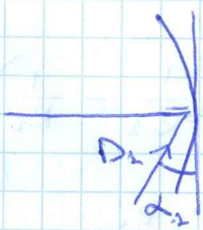
(1)




$$D_1 \cos\alpha_1 = D_2 \cos\alpha_2$$

$$\frac{D_1}{\varepsilon_0\varepsilon_1}\sin\alpha_1 = \frac{D_2}{\varepsilon_0\varepsilon_2}\sin\alpha_2$$

(2):




$$D_1 \sin\alpha_1 = \sigma \qquad D_2 \sin\alpha_2 = \sigma$$

$$\frac{D_1}{\varepsilon_1\varepsilon_0}\cos\alpha_1 = 0 \qquad \frac{D_2}{\varepsilon_2\varepsilon_0}\cos\alpha_2 = 0$$

$$\Rightarrow \alpha_1 = \alpha_2 = \pi/2$$

$\Rightarrow$ поле радиально. Воспользуемся т. Гаусса:

$$\oint D\,dS = Q \quad ; \; r \in [R_1; R_2]$$

$$\varepsilon_1\varepsilon_0 E \psi_1 \cdot r \cdot l + \varepsilon_2\varepsilon_0 E \psi_2 r \cdot l = Q$$

$$E = \frac{Q}{\varepsilon_0 l r(\varepsilon_1\psi_1 + \varepsilon_2\psi_2)}$$

$$E = -\frac{\partial \varphi}{\partial r} \Rightarrow \varphi = \frac{-Q}{\varepsilon_0 l\,(\varepsilon_1 \psi_1 + \varepsilon_2 \psi_2)} \ln r$$

$$U = \varphi_1 - \varphi_2 = \frac{Q}{\varepsilon_0 l\,(\varepsilon_1 \psi_1 + \varepsilon_2 \psi_2)} \ln \frac{R_2}{R_1}$$

$$C = \frac{Q}{U} = \frac{\varepsilon_0 l\,(\varepsilon_1 \psi_1 + \varepsilon_2 \psi_2)}{\ln R_2/R_1}$$

$\boxed{5.23}$ из 4.23 известно, что

$$E = \frac{q}{\varepsilon_0 r^2} \frac{1}{\Omega_1 \varepsilon_1 + \Omega_2 \varepsilon_2} = -\frac{\partial \varphi}{\partial r}$$

$$\varphi = \frac{q}{\varepsilon_0 r} \frac{1}{\Omega_1 \varepsilon_1 + \Omega_2 \varepsilon_2}$$

$$U = \varphi_1 - \varphi_2 = \frac{q}{\varepsilon_0 (\Omega_1 \varepsilon_1 + \Omega_2 \varepsilon_2)} \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$C = \frac{q}{U} = \frac{\varepsilon_0 (\Omega_1 \varepsilon_1 + \Omega_2 \varepsilon_2) R_1 R_2}{R_2 - R_1}$$